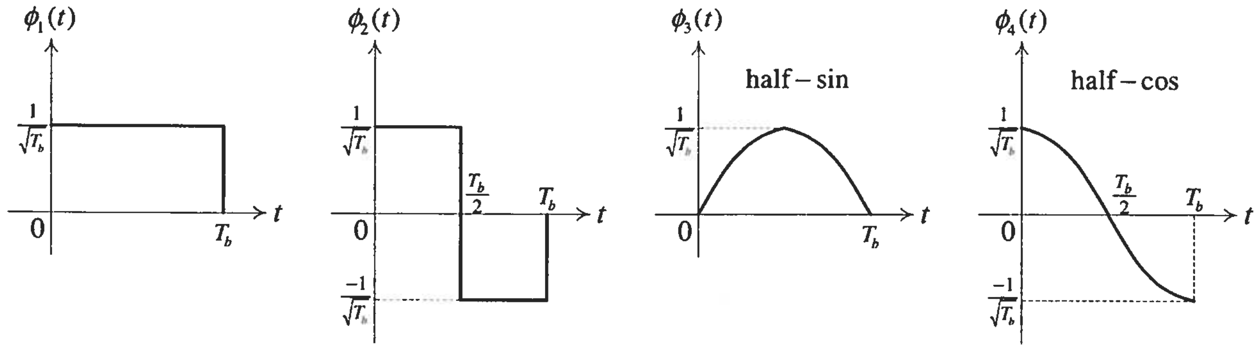
[2] (a) The figure below shows four functions, $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$ and $\phi_4(t)$.



It can be verified that $\phi_1(t)$ and $\phi_2(t)$ forms a set of *orthonormal* basis functions. **Clearly list <u>three</u> other pairs of *orthogonal* functions.**

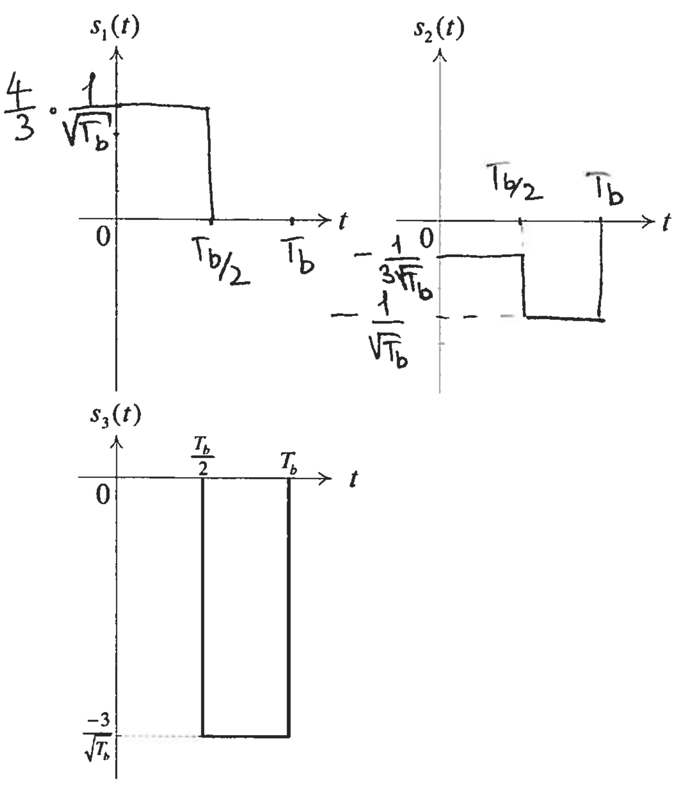
By inspection (using even/odd properties) : $\{\phi_1(t), \phi_4(t)\}$, $\{\phi_2(t), \phi_3(t)\}$, $\{\phi_3(t), \phi_4(t)\}$

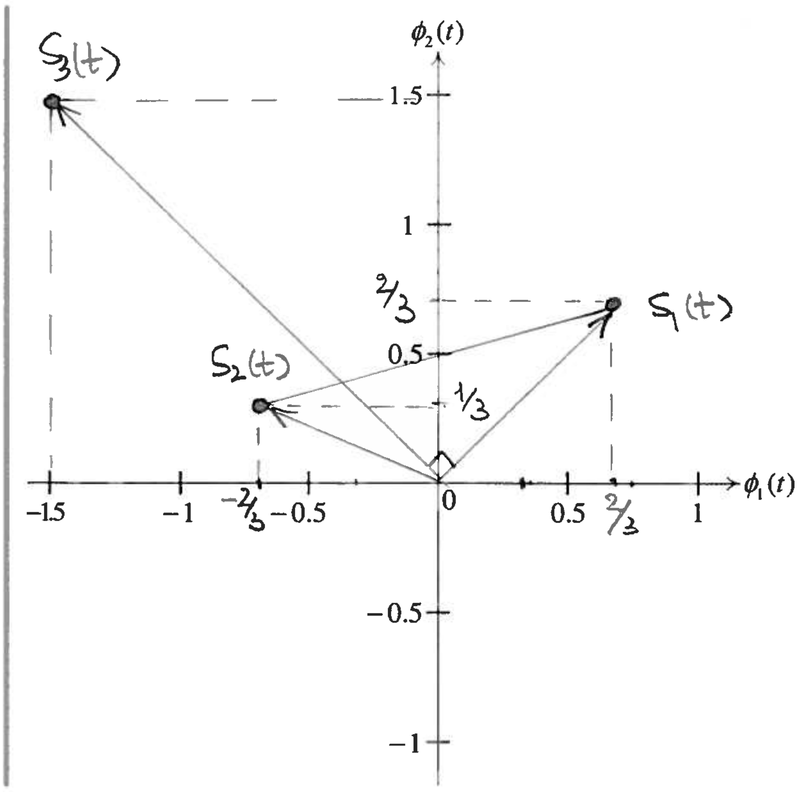
[4] (b) Using orthonormal functions $\phi_1(t)$ and $\phi_2(t)$ to construct two signals as follows:

$$s_1(t) = \frac{2}{3}\phi_1(t) + \frac{2}{3}\phi_2(t), \quad s_2(t) = -\frac{2}{3}\phi_1(t) + \frac{1}{3}\phi_2(t).$$

Obtain and neatly plot the two waveforms $s_1(t)$ and $s_2(t)$. Also clearly show the locations of the two waveforms (i.e., the tips of the corresponding signal vectors) on the signal space diagram spanned by $\{\phi_1(t), \phi_2(t)\}$. Compute the distance between $s_1(t)$ and $s_2(t)$.

$$d_{12}^2 = \left(\frac{1}{3}\right)^2 + \left(\frac{4}{3}\right)^2 = \frac{17}{9} \Rightarrow d_{12} = \frac{\sqrt{17}}{3}$$





[2] (c) Next, consider signal $s_3(t)$ as shown above. It is known that $s_3(t)$ can also be represented as a linear combination of $\phi_1(t)$ and $\phi_2(t)$. **Obtain such a combination and clearly show the location of $s_3(t)$ on the same signal space diagram in Part (b).**

$S_{31} = \int_0^{T_b} s_3(t)\phi_1(t)\,dt = -3/2$; $S_{32} = \int_0^{T_b} s_3(t)\phi_2(t)\,dt = \frac{3}{2}$

$\Rightarrow S_3(t) = -\frac{3}{2}\phi_1(t) + \frac{3}{2}\phi_2(t)$

[2] (d) **<u>From the signal space diagram</u>, determine the energy of signal $s_3(t)$. Also determine whether $s_3(t)$ is <u>orthogonal</u> to $s_1(t)$ and/or $s_2(t)$.**

$E_3 = 1.5^2 + (-1.5)^2 = \frac{9}{2} = 4.5$ joules ; $S_3(t)$ is orthogonal to $S_1(t)$, but not to $S_2(t)$